Engineering Notes

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Effect of Centrifugal Force on Range of the Aero-Space Plane

Charles W. Bert*
University of Oklahoma, Norman, Oklahoma

Nomenclature

C_0, C_1	= thrust coefficients of fuel consumption defined in Eq. (12)
C_D, C_L	= total drag and lift coefficients
C_{D0}, C_{D1}	= parasite-drag coefficient at constant M and at $M=1$
c	= $\frac{1}{2}M_m^3$
D	= total aerodynamic drag force
E, F_c	= endurance at cruise, centrifugal force
g, g_e	= gravitational acceleration at altitude and at Earth's surface
H	= true altitude above spherical Earth
K, K', K_1	= induced-drag factor at constant M, its effective value, and at $M=1$
L	= aerodynamic lift force
M	= Mach number in general
M_o	= orbital Mach number
M_m	= Mach number at which thrust specific fuel consumption is a minimum
m	= vehicle gross mass
q	= dynamic pressure
R, R_s	= cruising range, specific range
R_e, R_0	= Earth surface radius and R_e+H (radius from Earth's center)
S	= wing area
T, t	= thrust force and time
V, V_r	= absolute and relative velocities at cruise
V_a, V_e	= acoustic speed and Earth's peripheral velocity at the equator
$W, \dot{W}$	= vehicle gross weight and its time derivative
W_f, W_i	= final and initial values of W
θ	= latitude on Earth's surface
ρ	= atmospheric density at cruising altitude
ϕ	= flight-path direction measured from due east

Introduction

THERE is considerable engineering interest in the National Aero-Space Plane (NASP) or X-30, popularly known as the "Orient Express."[1] With cruising speeds that are a significant fraction of orbital speed, the effect of centrifugal force can no longer be ignored in the design of such a vehicle. This has been recognized by Etkin[2,3] and Bushnell.[4] However, to the best of the present investigator's knowledge, the specific impact of centrifugal force on the prediction of cruising range has been addressed in only one previous study by Drummond.[5] His analysis was limited to the case of flight along a minor-circle route.

The analysis follows closely the present investigator's previous analysis[6] for prediction of jet aircraft range at constant altitude. The historical aspects of the subject discussed in that paper are omitted here.

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*Professor of Engineering, School of Aerospace, Medical and Nuclear Engineering. Associate Fellow AIAA.

Basic Flight Mechanics

The total cruising range R can be expressed as

$$R = -\int_{W_f}^{W_f} R_s(W)\,\mathrm{d}W \tag{1}$$

where R_s is the specific range given by

$$R_s = -\frac{\mathrm{d}R}{\mathrm{d}W} = -V_r/\dot{W} \tag{2}$$

Here, V_r is the relative velocity (airspeed) and $\dot{W}$ the time rate of weight loss due to fuel consumption.

Assuming that the vehicle is in equilibrium at constant airspeed V_r in level flight at a radius R_0 from the center of the Earth (taken to be spherical), one can express the forces as

$$T = D = \tfrac{1}{2}C_D S\rho V_r^2 \tag{3}$$

$$W = L + F_c = \tfrac{1}{2}C_L S\rho V_r^2 + (mV^2/R_0) \tag{4}$$

where V is the absolute velocity and $W=mg$. Assuming a symmetric drag polar (for a fixed Mach number), one can write

$$C_D = C_{D_0} + KC_L^2 \tag{5}$$

In the subsonic flight regime, K is inversely proportional to aspect ratio. However, in supersonic flow, the effect of aspect ratio is much less pronounced[7] and reliable data for the hypersonic regime are almost nonexistent.

To study the effect of hypersonic flight velocity on specific range, it is convenient[8] to rewrite C_{D0} and K in Eq. (5) as

$$C_{D_0} = C_{D_1}/M, \quad K = K_1 M \tag{6}$$

where C_{D_1} and K_1 are constants and $M(=V_r/V_a)$ is the flight Mach number. Here, V_a is the speed of sound and V_r the relative velocity.

The absolute velocity V is the vector sum of the velocity of the Earth's surface at the latitude ϕ of interest ($V_e \cos\phi$) and the relative velocity or true airspeed $V_r(=MV_a)$. Thus,

$$V^2 = (V_e \cos\phi + MV_a \cos\theta)^2 + (MV_a \sin\theta)^2 \tag{7}$$

where V_e is the velocity of the Earth's surface at the equator and θ is the direction of the flight path, measured from the easterly direction. Then, the centrifugal force as a fraction of the weight is given by

$$F_c/W = (M^2 V_a + 2MV_a V_e \cos\theta \cos\phi + V_e^2 \cos^2\phi)/gR_0 \tag{8}$$

where

$$R_0 = R_e + H, \; g = g_e(R_e/R_0)^2$$

$$V_e = \frac{2\pi}{24 \text{ h} \times 3600 \text{ s/h}} R_e \tag{9}$$

Here, g_e is the gravitational acceleration at the Earth's surface, which is assumed to have a radius R_e.

As a numerical example, an altitude $H = 100{,}000$ ft (30,480 m) is used. From the ARDC model atmosphere[9] for that altitude, $V_a = 1005$ ft/s (306.3 m/s). Also, the mean radius of the Earth is[9] 3958.9 st. mi. or 20.90×10^6 ft (6370 km). The gravitational acceleration at the Earth's surface is 32.17 ft/s^2 (980.5 cm/s^2).

The importance of flight direction θ and latitude ϕ as well as Mach number is demonstrated in Table 1, which is based on Eq. (8) and the data mentioned. The drastic difference between eastward and westward flight is especially noteworthy.

Setting $F_c/W = 1$ in Eq. (8) leads to a quadratic equation in M, the solution of which affords a simple way to obtain the flight Mach number at which orbit is achieved M_o. Thus,

$$M_0 = \left[\left(\frac{V_e}{V_a}\right)^2 (\cos^2\theta - 1)\cos^2\phi + \left(\frac{gR_0}{V_a^2}\right)\right]^{1/2} \tag{10}$$

Now using Eqs. (9), one can rewrite Eq. (10) as

$$M_0 = [(V_e/V_a)^2 (\cos^2\theta - 1)\cos^2\phi + g_e(R_e + H)^{-1} \times (R_e/V_a)^2]^{1/2} - (V_e/V_a)\cos\theta\cos\phi \tag{11}$$

In Eq. (11), g_e, R_e, and V_e are fixed, while V_a is a function of altitude H. Thus, for orbital flight, at a fixed altitude, M_o depends upon flight parameters θ and ϕ. For example, for $H = 100{,}000$ ft (30,480 m), M_o varies from 24.23 for easterly flight at the equator to 25.69 for northerly flight at $\phi = 35$ deg.

In the present analysis, as in all known previous investigations, the effect of Coriolis force has been neglected. It can be shown that even under conditions most favorable to maximizing Coriolis force, it is considerably less than the centrifugal force for Mach numbers of 5 or greater.

Range and Endurance Prediction

To calculate the cruising range by integration of Eq. (1), one must have a cruising flight strategy as well as a propulsion fuel consumption model. In most range investigations for jet-propelled vehicles, it has been assumed that the thrust specific fuel consumption $-\dot{W}/T$ is a constant. Also, usually it has been assumed that, as the fuel is consumed, the thrust C_L/C_D and speed have all remained constant. Unfortunately, however, this set of assumptions is not internally consistent. A previous study[6] of turbojet/turbofan aircraft range by the present investigator removed these two limitations. Fuel consumption was taken to be $-\dot{W} = C_0 + C_1 T$ and the angle of attack was assumed to vary to permit C_L/C_D to change, as fuel was consumed, in accordance with the drag polar for specific weight and velocity.

It is generally recognized that the most feasible propulsion system for the Mach number range of the NASP is a scramjet.[1,5,10] Thus, the following scramjet fuel consumption model, based on the numerical predictions of Mordell and Swithenbank,[11] is used:

$$-\dot{W}/T = C_0 + C_l(M + \tfrac{1}{2}M_m{}^3/M^2) \tag{12}$$

where C_0 and C_1 are constants and M_m the Mach number corresponding to the minimum thrust specific fuel consumption. For the numerical data presented in Ref. 11, $M_m = 9$, $C_0 = -4.250$ h^{-1}, and $C_1 = 0.4471$ h^{-1}.

The cruising strategy used here is to hold altitude, thrust, and relative velocity V_r (and thus absolute velocity V) constant, while letting C_L/C_D vary as dictated by the drag polar for the vehicle. Thus, combining Eqs. (2–5) and (12), one obtains

$$R_s{}^{-1} = B_0 + B_1 W^2 \tag{13}$$

where

$$B_0 = \left(C_{D0} S \rho M \frac{V_a}{2}\right)[C_0 + C_1(M + cM^{-2})]$$

$$B_1 = \left(\frac{2K}{\rho S} M^3 V_a^3\right)[C_0 + C_1(M + cM^{-2})]\left[1 - \left(\frac{V^2}{gR_0}\right)\right] \tag{14}$$

Using Eqs. (13) and (14), one can express Eq. (1) in the same mathematical form as that obtained in Ref. 6, namely

$$R = -\int_{W_i}^{W_f} \frac{b\,\mathrm{d}W}{a^2 + W^2} \tag{15}$$

where

$$a = (B_0/B_1)^{1/2}, \; b = 1/B_1 \tag{16}$$

Equation (15) can be integrated to yield

$$R = (b/a)\,[\arctan(W_i/a) - \arctan(W_f/a)] \tag{17}$$

Since V_r is a constant, the endurance E is given by

$$E = R/V_r = (b/aV_r)\,[\arctan(W_f/a) - \arctan(W_f/a)] \tag{18}$$

Inspecting the parameters a and b in Eqs. (16), with B_0 and B_1 as given by Eqs. (14), one finds that the effect of centrifugal force is contained in the factor $(1 - V^2/gR_0)^2$. The way in which this factor appears in the equations is such that one can define an effective induced-drag factor as follows:

$$K' = K(1 - V^2/gR_0)^2 \tag{19}$$

It is noted that as V^2/gR_0 approaches unity, i.e., as orbital

Table 1 Centrifugal force as a percentage of weight at an altitude of 100,000 ft

Latitude θ Flt. direction θ Mach No.	0 deg (equator) 0 deg (E)	 15 deg	 180 deg (W)	35 deg (Tokyo) 0 deg (E)	 15 deg	 180 deg (W)
5	6.4	5.2	1.8	5.9	4.9	2.1
10	20.0	17.6	10.9	19.1	117.1	11.6
15	41.2	37.6	27.5	39.8	36.9	28.6
20	69.9	65.1	51.6	68.1	64.2	53.1
24	98.3	88.4	76.3	96.2	91.5	78.2

velocity is reached, lift L and thus C_L must vanish, in accordance with Eq. (4). In this special case, Eq. (13) reduces to

$$R_s{}^{-1}=B_0 \tag{20}$$

and Eqs. (15) is replaced by

$$R=-\left(\frac{1}{B_0}\right)\int_{W_i}^{W_f}\mathrm{d}W \tag{21}$$

which can be integrated to give

$$R=(W_i-W_f)/B_0 \tag{22}$$

Also,

$$E=R/V_r=(W_i-W_f)/B_0V_r \tag{23}$$

Application to Vehicle Optimization

To study the effect of hypersonic Mach number on the specific range, it is convenient to replace C_{D_0} and K as specified in Eq. (6). Then, for $V^2/gR_0<1$

$$R_s^{-1}=a_{-2}M^{-2}+a_{-1}M^{-1}+a_0+a_1M+\tfrac{1}{4}M^2+a_3M^3 \tag{24}$$

where the a are extensive algebraic expressions of the parameters.

At orbital speed $V^2/gR_0=1$ and

$$R_s^{-1}=B_0=\tfrac{1}{2}C_{D_1}S\rho V_a(C_0+C_lM+cC_1M^{-2})$$

or

$$R_s^{-1}=a_{-2}'M^{-2}+a_0'+a_1'M \tag{25}$$

where

$$a_{-2}'=\tfrac{1}{2}C_{D_1}C_1cS\rho V_a,\quad a_0'=\tfrac{1}{2}C_{D_1}C_0S\rho V_a$$
$$a_1'=\tfrac{1}{2}C_{D_1}C_1S\rho V_a \tag{26}$$

Setting $\mathrm{d}(R_s^{-1})/\mathrm{d}M=0$ yields

$$M_{\rm opt}=(2a_{-2}'/a_1')^{1/3}=(2c)^{1/3}=M_m \tag{27}$$

$$(R_s)_{\rm opt}=a_{-2}'M_m^{-2}+a_0'M_m \tag{28}$$

Equation (27) shows that the optimal Mach number to achieve maximum range in orbit is M_m, the Mach number at which the $-\dot{W}/T$ is a minimum.

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Optimum Structural Sizing for Gust-Induced Response

P. Hajela* and C. T. Bach†
University of Florida, Gainesville, Florida

Introduction

THE use of mathematical nonlinear programming algorithms has enjoyed considerable success in the automated structural synthesis environment. A majority of research pertaining to optimum structural design has focused primarily on statically loaded structures, and more effort needs to be directed at developing sizing capabilities for dynamic loads, in particular, nondeterministic loads.[1] The optimum sizing of airfrace structures requires an analysis tool that accounts for static and dynamic structural stability, deformations under applied loads, and the interaction of structural deformations and airloads.

The present work was directed toward establishing an optimization capability for sizing wing structures that are subjected to a combination of deterministic and random flight loads. This included the implementation of efficient methods for computing response sensitivity required by the optimization algorithm. The random loads were treated as a stationary, homogeneous process with a Gaussian distribution. A frequency-domain analysis was selected for the solution of the dynamic response problem, wherein the gust loads were represented by a power spectral density spectrum of gust velocities.

For a structure subjected to nondeterministic loads, failure can result either from a single exceedance of stress, or from cumulative damage due to fatigue. For these failure modes, Johnson[2] formulates design constraints applicable only when a single stress component is considered critical in the constraint definitions. Constraints for fatigue failure have also been obtained from a fracture mechanics standpoint.[3] If a combination of the response quantities is involved, as in the

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*Associate Professor, Aerospace Engineering, Mechanics, and Engineering Science. Member AIAA.

†Graduate Research Assistant, Aerospace Engineering, Mechanics, and Engineering Science. Student Member AIAA.